Introduction

It is a common observation that teaching methods appear to cycle from formal to informal, from didactic teaching to more creative approaches. The new curriculum for maths may seek to emphasise the subject's interconnectedness and creativity but there is an evident focus upon calculation within the primary phase. Establishing such functionality at an early stage provides the necessary tools for children to begin the exploration of number and pattern. This book provides examples of key strategies within calculation.

Mathematics is sometimes thought of as a Marmite subject which children either love or loathe. This is a view echoed within the general population; indeed it seems to be socially acceptable to admit one's inability in maths. I have lost count of how many parents have stated quite candidly during open evenings, 'Oh, I'm just hopeless at maths,' or 'I couldn't do maths at school'. Since no one has ever made the same admission about reading or writing it must be assumed that maths is a subject in which failure carries little stigma.

Boaler (2009: 95) states: 'The idea that only some children can do well in maths is deeply cultural and it exists in England and the United States. It is also extremely harmful to children's learning and it is something that we need to change. Every child can do as well in maths as they do in other subjects – if they receive good teaching and people believe in them'. It makes sense that we should all have some innate ability in mathematics. Not only are we genetically programmed to identify sensory patterns but the recognition of more and less, and the ability to add and subtract simple numbers, are obvious life skills in any environment. That every child has the capacity to learn mathematics, and ultimately wishes to do so, is a conviction that is as challenging to hold as it is fundamental to successful teaching.

I have taught maths to primary school children for decades but it was teaching on an individual basis that allowed me to realise just what mathematical demons some children carry with them into numeracy lessons. I gained insights into teaching that would have been unavailable within a class setting and duly share these findings below:

Some children will have learned to anticipate failure

When children have learned to fail in mathematics they will attribute that failure in one of two ways: either maths is stupid or they are stupid. In the latter instance children can be remarkably resilient in retaining the belief despite obvious evidence to the contrary. Pages of their own correct workings are dismissed as irrelevant and they may refuse to participate in any celebration of their achievement. One incorrect answer is seen as irrefutable proof that they can't 'do' maths – 'See? I *told* you I couldn't do it!' Self-diagnosed stupidity becomes a comfortable get-out clause that condones apathy. It is a long road to recovery from this position, so early feelings of success are vital.

Children are good at seeming to cope

When a teacher is faced with a bewildering array of children in a maths class it is tremendously difficult to identify which child really needs support. Some children ask for help for simple reassurance, some ask for help because it's easier to be helped than to think for oneself, and most do so because they have the wherewithal to understand that asking for help is a proactive part of learning. There are inevitably some children who do not ask for help: they appear to cope and consequently fade surreptitiously into the background. Teachers should not underestimate the sophistication and brilliance of children's coping strategies certain of which will be to divert attention to others. Just because a child appears competent in an area of maths does not mean that they have any understanding of what they are doing.

Small steps versus big jumps

Steve Chinn (2004: 59) identifies two main types of mathematical thinking and characterises them as grasshoppers and inchworms. Grasshopper students have an apparently intuitive ability to solve problems and present their own solutions, often without recourse to recording. Conversely, inchworm students edge their way towards an answer following set procedures and documenting their progress. Determining a child's thinking style then supporting his strengths and challenging his weaknesses is a vital element within successful teaching. Chinn (2004: 71) also cautions: 'Teachers need to realistically appraise their own thinking style when teaching maths and appraising maths and look at the pupils who sail through their lessons. Then they should look at the pupils who struggle and see if a mismatch of thinking style is a contributory factor.' For many teachers the small steps approach can be tedious even frustrating, but it can offer a lifeline to pupils especially with the recognition that, for some, no step is too small.

Mathematical thinking is a specific skill which can be recognised and taught

Children can be taught to identify when they are thinking mathematically. They are able to recognise when they are in a mathematical 'zone' and discern when their concentration has lapsed. Teaching metacognition within primary schools may seem challenging but children are oddly comfortable with the idea. Developing an understanding of their own thought processes provides a starting point for the sharing and discussion of ideas and methods.

Talking is essential

Children need opportunities to share their methods. They can invent the most incredibly convoluted ways of arriving at answers but in doing so they express their personal understanding of mathematics. Teachers may wish to redefine children's ideas for class consumption but must always value the original process.

Successful mathematicians welcome failure

The most vibrant students enjoy being challenged. Not achieving an answer on the first try becomes a spur to further efforts and increases motivation to ask questions. All children ultimately need to feel that they have succeeded and skilled teaching will provide just enough support to permit the belief that they have discovered a solution by themselves. Challenge and support are the key elements in developing a healthy attitude to maths work.

Rote learning can be positive and rewarding

The focus on critical thinking, upon pupils developing an understanding of mathematical processes, has consigned rote learning to becoming a feature of poor teaching and 'unsatisfactory' lessons. Mathematical algorithms are deliberately designed to be easy to implement; they save us time and effort but the mathematics of these algorithms can be considerably more complex than their process. Perversely, we traditionally insist that children should have a critical understanding of the method before the algorithm is taught. It needs to be acknowledged that, in some instances, it is acceptable for children to learn how to perform an algorithm by rote then, in due course, to apply critical thinking to the mechanics of the processes. Zukav (1984: 208) quotes John von Neumann, one of the foremost mathematicians of the 20th Century as saying: 'Young man, in mathematics you don't understand things. You just get used to them.' For some pupils managing their learning in this way feeds success and positive attitudes.

Not all mathematics activities need to have teacher-led challenge and purpose

Curriculum pressures mean that teachers are obliged to push children forward in their work. Every lesson has a learning objective, success criteria and targets. Where, one wonders, is the opportunity for reinforcement and, more controversially, the opportunity for what might be termed 'Maths-Lite'? Ruth Merttens (1996: 53) suggests that learning mathematics has similarities to learning a language; it is interesting to continue the analogy. When children finish reading a book at school they will receive a more challenging text, but at home they may elect to settle down with a familiar book, a book in which the vocabulary is accessible and the outcome already known. Comfort reading, as this is known, is an important aspect of learning as the child gains confidence and pleasure in the process of repetition. It is unlikely for children to engage in formal mathematics at home thus they are denied similar opportunities for learning. A few easy calculations every once in a while can boost confidence and make mathematics a more friendly subject to those children who might be feeling pressured by the pace of teaching.

Finally, a plea: streaming in primary mathematics is a common strategy to aid differentiation. Effective teachers will keep their groups under constant review. Mathematics is a broad church: children can be outstanding at calculation and utterly confused with data handling or geometry. It is therefore appropriate that differentiation should depend upon the child's ability within the subject matter rather than the group in which they have been placed in an unrelated aspect of maths.

Using this book

Each section within this book begins with Teacher's notes followed by photocopiable activity sheets. The activities are presented within the context of a robot world (the 'Robosphere'), with an introductory sheet establishing this theme and the digibot motifs used throughout.

ACTIVITY

The Teacher's notes provide further information about methodology and contextualise activities as necessary.

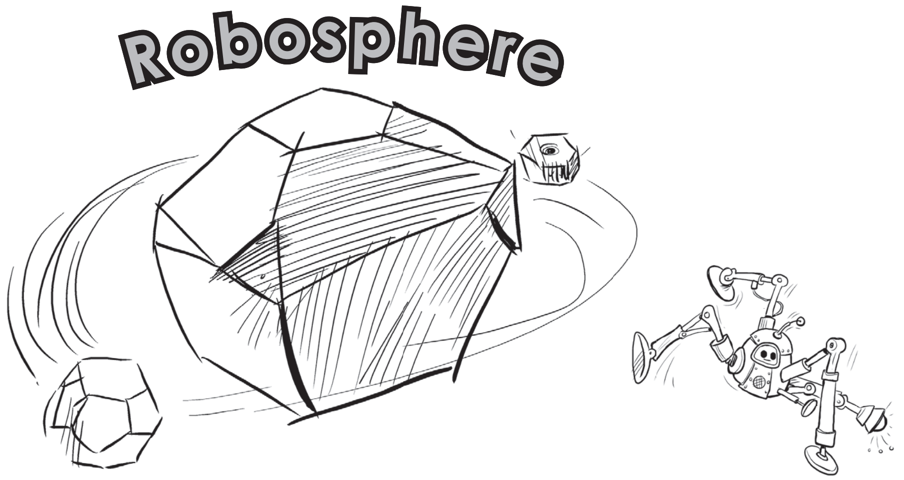
The Activity sheets are described and explained beneath the Teacher's notes and are indicated by the banner on shown on the left.

Resources

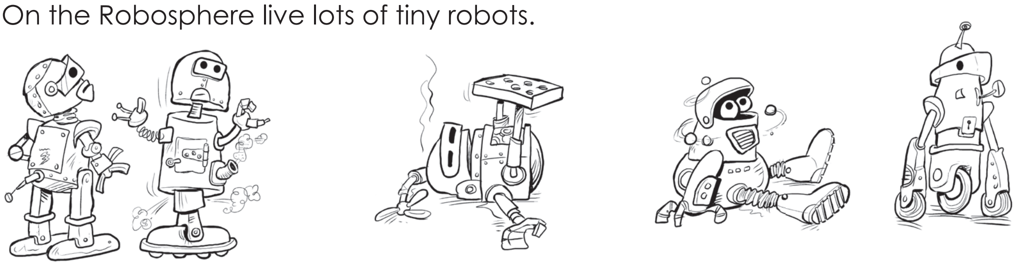
ACTIVITY 01

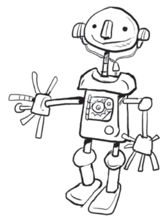
In the far reaches of space an extraordinary world spins slowly around an impossible sun.

This world is the . . .



On the Robosphere live lots of tiny robots.

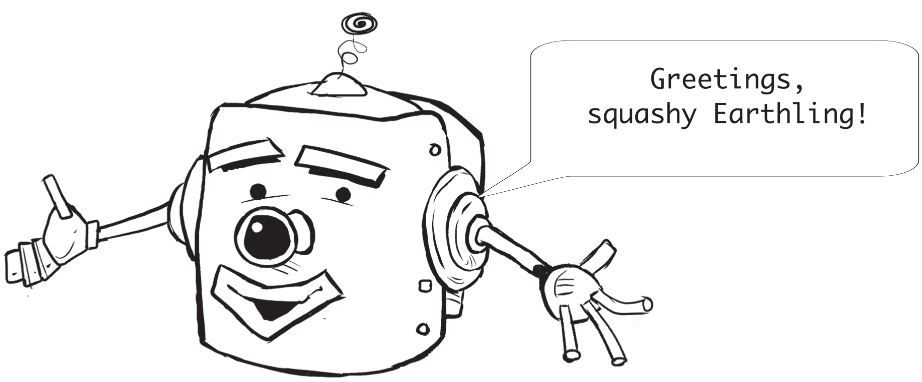




Some of these are called digibots. Digibots clank and jangle themselves together to make numbers.

Sometimes you'll see robots who point out examples of how calculations have been worked out.

The digibots are controlled by the Big Robot Head, sometimes known just as 'Big Head'.



Place value and decimal places

Introduction

Pupils in year 5 are expected to read, write, order and compare numbers to at least 1 000 000; for year 6 pupils this is extended to 10 000 000.

As numbers get larger they become increasingly abstract. For pupils of upper primary age, numbers larger than one thousand can be difficult to contemplate in real terms and a million can become a notional description of a big number rather than a specific amount. It is therefore worthwhile spending time investigating the physicality of big numbers in terms of their length, height, area, volume and time. Pupils can be asked to research and calculate a range of everyday items, for example how many blades of grass are on their field or leaves on a tree; how many words in a book or pages in their library; how many people live in their community? Similarly, units in different dimensions can be difficult to rationalise: a million centicubes (or Dienes' units) laid end to end will stretch to a distance of ten kilometres but can be placed into a square of just ten metres; stacked into a neat cube they will measure just one metre in each dimension. Gaining an understanding of quantity, dimension and scale is essential if calculations are to be imbued with any meaning beyond the practising of an algorithm.

Place value, of course, extends to decimal places. Most pupils will have gained some familiarity in using decimal notation for money but may not have extended their thinking to understand decimals as fractions of units. For this reason the section on place value includes activities on decimal fractions.

ACTIVITIES

Place value (1): ten thousand
There are ten thousand dots on this activity sheet. Pupils could be asked to imagine this as representing different items. How much space would ten thousand people need to stand in this configuration? How long would it take them to eat ten thousand evening meals? How big a car park would be needed to contain ten thousand cars?

Place value (2)
This sheet presents a list of facts to help understand the enormity of one million within understood concepts. Pupils are invited to contribute their own information.

Place value (3)
This sheet focuses upon the effect of place value when multiplying tens, hundreds and thousands.

Place value (4)
Pupils are asked to write the values shown in the digibots. The bottom section is offered for reference but may be deleted from the page during copying.

Place value (5) and resource sheet
Pupils have to cut out the numbers from the resource sheet and identify their position to make the numbers listed. Two sets of cards are included to avoid neighbouring pupils mixing up their resources.

Place value game
The teacher draws from a pack of ten single digit cards and reads out the digit. The children then need to position the digits correctly on their sheet to make the highest number possible for addition then subtraction calculations.

Place value (6)
This activity sheet shows the structure implicit with multiplication and division place values, with calculations that develop in complexity. The second section comprises decimals and can be used with the place value and digibot sheet (on the following pages).

Place value and digibot sheets
The digibots are designed to be copied onto clear acetates. Pupils can then write numbers onto the digibot bodies and see the place values change as they are moved across the place value sheets. The place value sheet contains a row for whole numbers, and one with decimals to three digits. Children should aim to work without these resources as soon as they feel confident to visualise the movement of digits.

Ordering decimals (1)
Ordering numbers with one and two decimal places. Most of the numbers can be sorted by whole values.

Ordering decimals (2)
The activity from Ordering decimals (1) is repeated but the focus now requires a greater consideration of the value of digits beyond the decimal point.

Comparing decimals
This is a third decimal sorting sheet, but here the pupils are directed to give the numbers the same amount of decimal places. Teachers should assure that pupils understand that adding a zero to the right of the decimal digit does not change its value.

ACTIVITIES

Calculating with decimals

An explanation and activity sheet with examples for addition and subtraction of decimals with different decimal places.

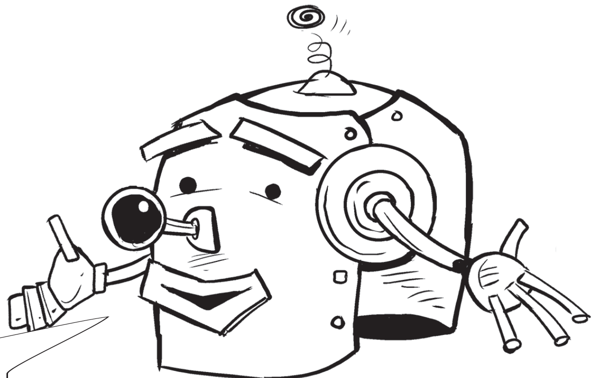
Decimal sorting activity

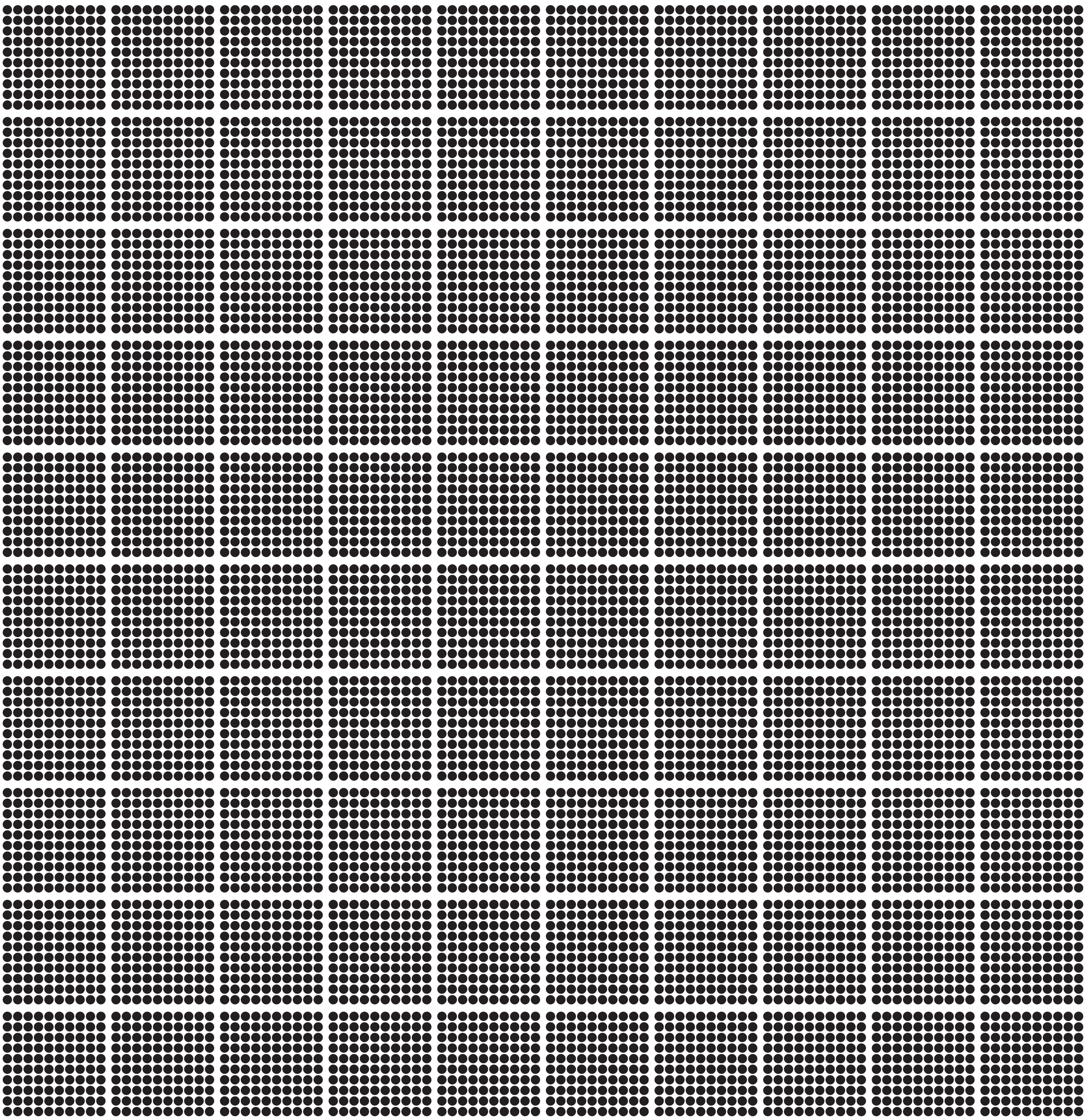
A template for pupils to fill in their own decimal numbers. This can be completed individually or as a group. Once completed the numbers are cut out then swapped with another set. Individuals or groups race to place the decimal cards in the correct order. A similar activity can be completed as a class with children 'wearing' Post-it notes of decimal numbers. It becomes all the more interesting if the Post-it notes are positioned on their backs and the activity is conducted in silence…

Decimal place value game (1) and (2)

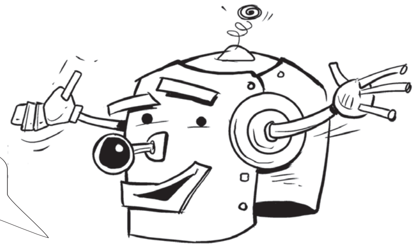
As with the first place value game, the teacher draws a card from a pack of ten single digit cards. Pupils position each digit in the correct place to achieve the highest or lowest target number. In each game one digit may be discarded to the robot bin. An alternative game is to replace a digit card each time it is drawn. This alters the probabilities and more able children will incorporate this fact in their deliberations.

Place value (1)





Place value (2)

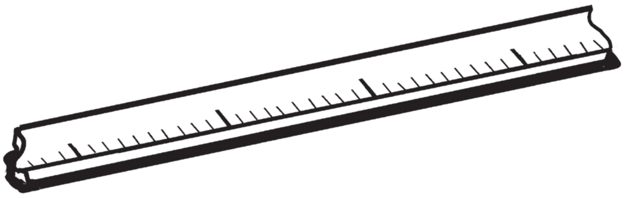


One million is a BIG number!

1 000 000

One million is the number of . . .

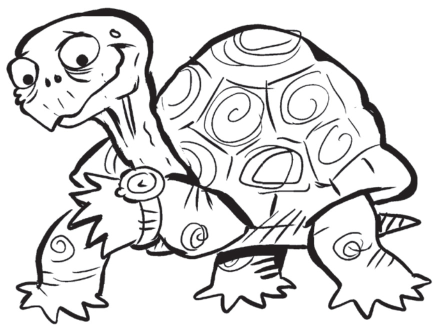
millimetres in a kilometre



millilitres in 4 000 cups of tea

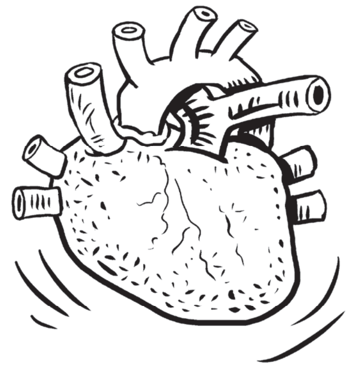
seconds in 11½ days

hours in 114 years



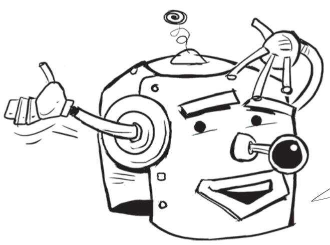
words you would write in a year if you wrote 2 740 words every day!

times your heart beats in 9 ½ days



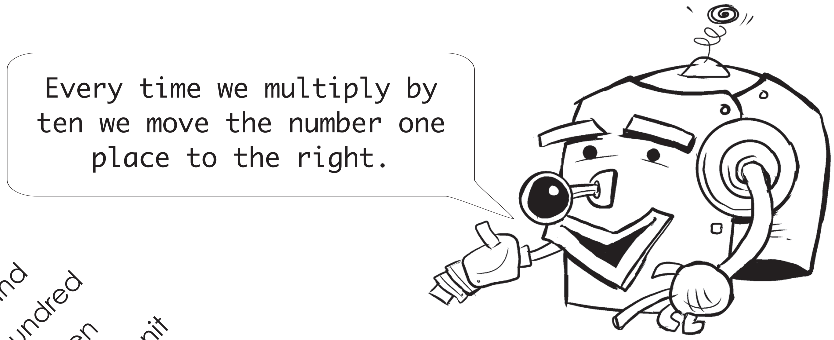
pixels on a computer screen (1 280 x 800 has a resolution of 1 024 000 pixels)

and finally… it would take you 3 846 years of saving £5 every week before you had £1 000 000.



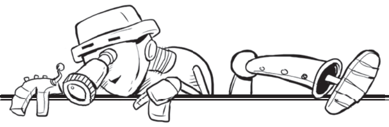
How many more facts about one million can you find or calculate?

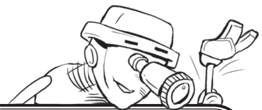
Place value (3)



ten million	million	hundred thousand	ten thousand	thousand	hundred	ten	unit
							1
						1	0
					1	0	0
				1	0	0	0
			1	0	0	0	0
		1	0	0	0	0	0
	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0

10 x 1
10 x 10
10 x 10 x 10
10 x 10 x 10 x 10
10 x 10 x 10 x 10 x 10
10 x 10 x 10 x 10 x 10 x 10
10 x 10 x 10 x 10 x 10 x 10 x 10





10 x 100 = 1 000
1 zero + 2 zeros = 3 zeros
one thousand

100 x 100 = 10 000
2 zeros + 2 zeros = 4 zeros
ten thousand

Now try these:

a) 1 000 x 1 000 = ________________
3 zeros + 3 zeros = 6 zeros
____

b) 100 x 1 000 = ________________
2 zeros + 3 zeros = 5 zeros
____

c) 10 x 100 x 1 000 = ________________
____

d) 10 x 100 x 10 = ________________
____

e) 10 x 10 x 10 x 10 = ________________
____

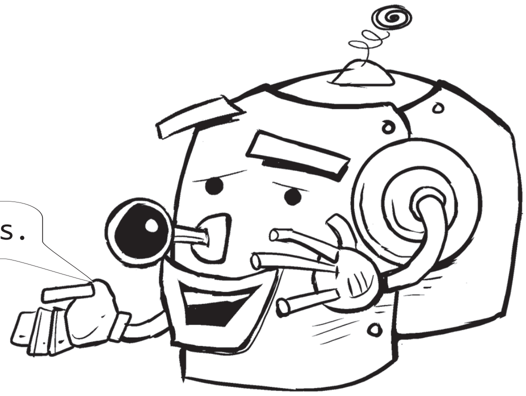
f) 100 x 100 x 100 = ________________
____

g) 1 000 x 10 x 10 x 100 = ________________
____

h) 100 x 10 x 100 = ________________
____

Place value (4a)

Write out the numbers below in words.



a) twelve thousand, seven hundred and ninety-five

b) ______________________________

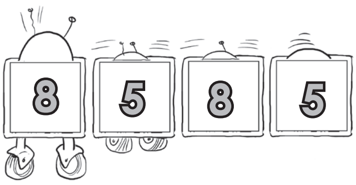
____

c) ______________________________

____

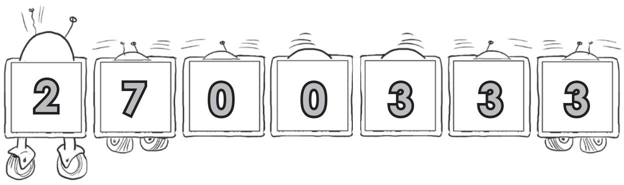
d) ______________________________

____



e) ______________________________

____



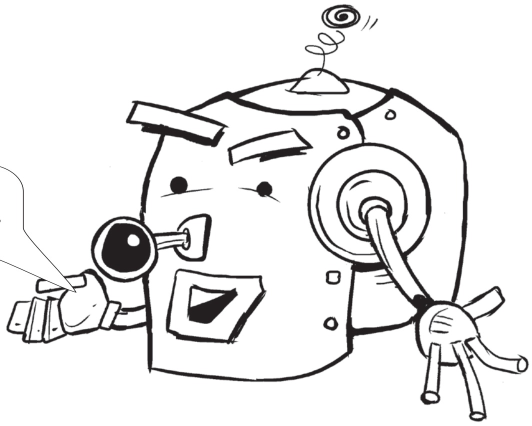
f) ______________________________

____

one	two	three	four	five	six	seven
eight	nine	ten	twenty	thirty	forty	fifty
sixty	seventy	eighty	ninety	hundred	thousand	million

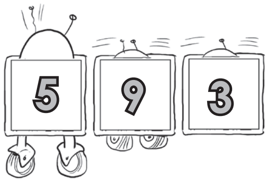
Place value (4b)

Write out the numbers below in words.



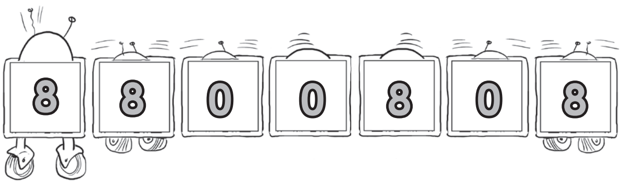
g) ____________________

____



h) ____________________

____



i) ____________________

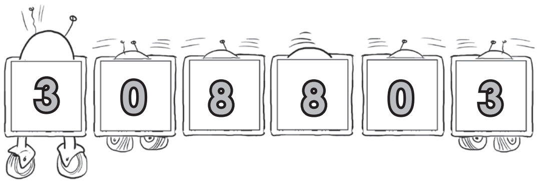
____

j) ____________________

____

k) ____________________

____



l) ____________________

____

one	two	three	four	five	six	seven
eight	nine	ten	twenty	thirty	forty	fifty
sixty	seventy	eighty	ninety	hundred	thousand	million